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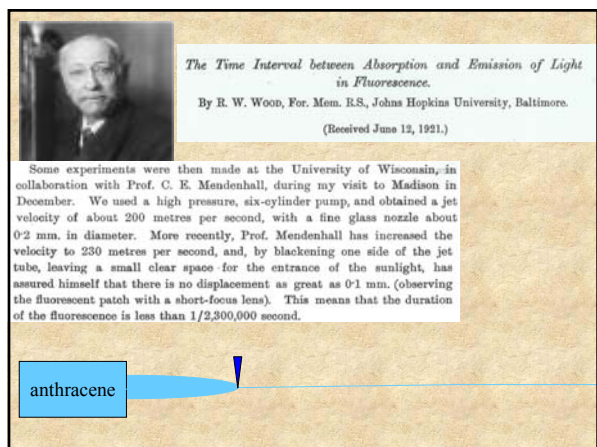
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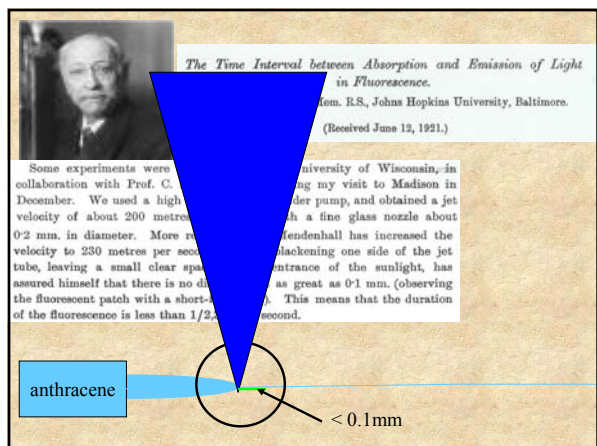
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*The Time Interval between Absorption and Emission of Light in Fluorescence.*

By R. W. Wood, For. Mem. R.S., Johns Hopkins University, Baltimore.

(Received June 12, 1921.)

Some experiments were then made at the University of Wisconsin, in collaboration with Prof. C. E. Mendenhall, during my visit to Madison in December. We used a high pressure, six-cylinder pump, and obtained a jet velocity of about 200 metres per second, with a fine glass nozzle about 0.2 mm. in diameter. More recently, Prof. Mendenhall has increased the velocity to 230 metres per second, and, by blackening one side of the jet tube, leaving a small clear space for the entrance of the sunlight, has assured himself that there is no displacement as great as 0.1 mm. (observing the fluorescent patch with a short-focus lens). This means that the duration of the fluorescence is less than  $1/2,300,000$  second.

anthracene

i.e. < 435ns

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This work was followed by a report by Philip Gottling in 1923 who used a Kerr Cell – as originally suggested by Lord Rayleigh in 1905.

THE DETERMINATION OF THE TIME BETWEEN EXCITATION AND EMISSION FOR CERTAIN FLUORESCENT SOLIDS

By PHILIP F. GOTTLING

ABSTRACT

Time lag between excitation and emission of fluorescence by barium platino-cyanide and rhodamine.—The work begun in 1921 by R. W. Wood on the measurement of fluorescent intervals and phosphorescent times has been continued. The method of Abraham and Lemoine, somewhat modified, was used for determining the very short periods of time involved. The fluorescent light is polarized and then passed through a condenser, containing nitrobenzene as dielectric, which had begun to be discharged when the illuminating spark started. The later the light arrives the lower the average field of the condenser and the smaller the angular setting of the analyzing Nicol to match the two images produced by a double image prism. The apparatus was calibrated by means of light reflected from a mirror at different distances from the spark. The interval of time between the occurrence of a spark and the emission of the fluorescent light excited by that spark, was found to be  $(2.12 \pm .01) \times 10^{-9}$  sec. for barium platino-cyanide and  $(2.11 \pm .14) \times 10^{-9}$  sec. for rhodamine.

He found 21.1 ns for a solution of rhodamine in acetone, acetic acid and glycerol. Possibly he was observing a combination of fluorescence and phosphorescence

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Ein Fluorometer.  
Apparat zur Messung von Fluoreszenzabklingungszeiten.

Von E. Gaviola in Berlin.

Mit 9 Abbildungen. (Eingegangen am 24. März 1927.)

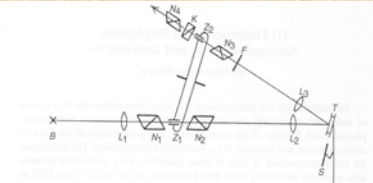


FIG. 1. Original apparatus of Gaviola' for the measurement of fluorescence lifetimes, described in text. B, Source of exciting light; T, cuvette containing the fluorescent solution; S, mirror.

Enrique Gaviola

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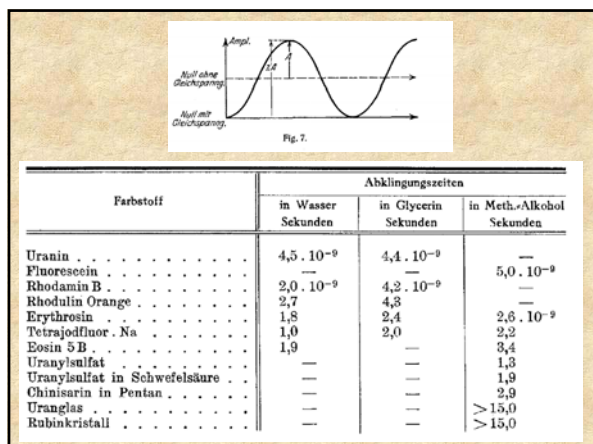
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What is meant by the "lifetime" of a fluorophore???

Although we often speak of the properties of fluorophores as if they are studied in isolation, such is not usually the case.

Absorption and emission processes are almost always studied on *populations* of molecules and the properties of the supposed typical members of the population are deduced from the macroscopic properties of the process.

In general, the behavior of an excited population of fluorophores is described by a familiar rate equation:

$$\frac{dn^*}{dt} = -n^* \Gamma + f(t)$$

where  $n^*$  is the number of excited elements at time  $t$ ,  $\Gamma$  is the rate constant of emission and  $f(t)$  is an arbitrary function of the time, describing the time course of the excitation. The dimensions of  $\Gamma$  are  $\text{sec}^{-1}$  (transitions per molecule per unit time).

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If excitation occurs at  $t = 0$ , the last equation, takes the form:

$$\frac{dn^*}{dt} = -n^* \Gamma$$

and describes the decrease in excited molecules at all further times. Integration gives:

$$n^*(t) = n^*(0) \exp(-\Gamma t)$$

The lifetime,  $\tau$ , is equal to  $\Gamma^{-1}$

If a population of fluorophores are excited, the lifetime is the time it takes for the number of excited molecules to decay to  $1/e$  or 36.8% of the original population according to:

$$\frac{n^*(t)}{n^*(0)} = e^{-t/\tau}$$

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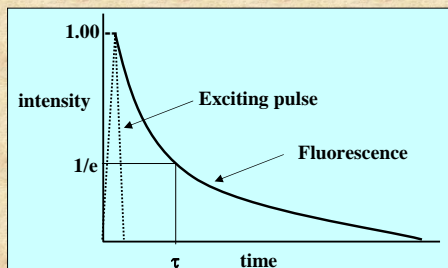
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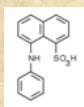
In pictorial form:

$$\frac{n^*(t)}{n^*(0)} = e^{-t/\tau}$$

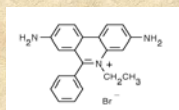


The lifetime and quantum yield for a given fluorophore is often dramatically affected by its environment.

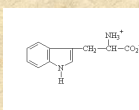
Examples of this fact would be NADH, which in water has a lifetime of ~0.4 ns but bound to dehydrogenases can be as long as 9 ns.



ANS in water is ~100 picoseconds but can be 8 – 10 ns bound to proteins



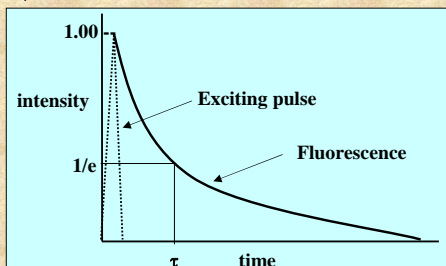
Ethidium bromide is 1.8 ns in water, 22 ns bound to DNA and 27ns bound to tRNA



The lifetime of tryptophan in proteins ranges from ~0.1 ns up to ~8 ns

Excited state lifetimes have traditionally been measured using either the *impulse* response or the *harmonic* response method. In principle both methods have the same information content. These methods are also referred to as either the "time domain" method or the "frequency domain" method.

In the *impulse* (or pulse) method, the sample is illuminated with a short pulse of light and the intensity of the emission versus time is recorded. Originally these short light pulses were generated using *flashlamps* which had widths on the order of several nanoseconds. Modern laser sources can now routinely generate pulses with widths on the order of picoseconds or shorter.

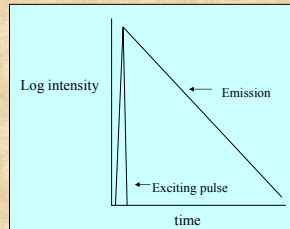


As shown in the intensity decay figure, the *fluorescence* lifetime,  $t$ , is the time at which the intensity has decayed to  $1/e$  of the original value. The decay of the intensity with time is given by the relation:

$$I_t = \alpha e^{-t/\tau}$$

Where  $I_t$  is the intensity at time  $t$ ,  $\alpha$  is a normalization term (the pre-exponential factor) and  $\tau$  is the lifetime.

It is more common to plot the fluorescence decay data using a logarithmic scale as shown here.




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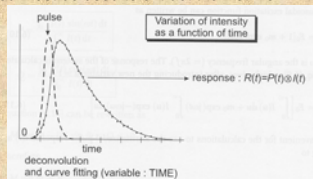
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If the decay is a single exponential and if the lifetime is long compared to the exciting light then the lifetime can be determined directly from the slope of the curve.

If the lifetime and the excitation pulse width are comparable some type of *deconvolution* method must be used to extract the lifetime.



Great effort has been expended on developing mathematical methods to "deconvolve" the effect of the exciting pulse shape on the observed fluorescence decay.

With the advent of very fast laser pulses these deconvolution procedures became less important for most lifetime determinations, although they are still required whenever the lifetime is of comparable duration to the light pulse.

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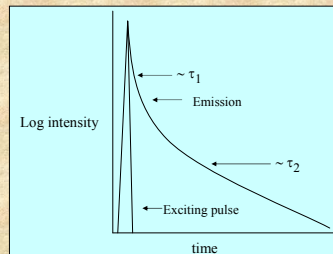
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If the decay is multiexponential, the relation between the intensity and time after excitation is given by:

$$I(t) = \sum_i \alpha_i e^{-t/\tau_i}$$

One may then observe data such as those sketched below:



Here we can discern at least two lifetime components indicated as  $\tau_1$  and  $\tau_2$ . This presentation is oversimplified but illustrates the point.

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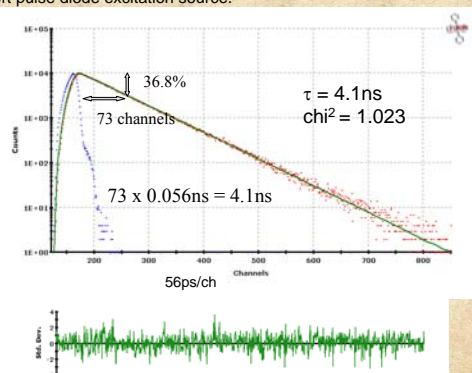
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Here are pulse decay data on anthracene in cyclohexane taken on an IBH 5000U Time-correlated single photon counting instrument equipped with an LED short pulse diode excitation source.




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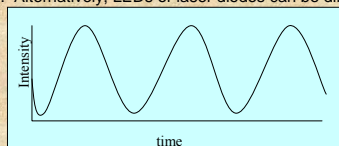
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In the harmonic method (also known as the phase and modulation or frequency domain method) a continuous light source is utilized, such as a laser or xenon arc, and the intensity of this light source is modulated sinusoidally at high frequency as depicted below. Typically, an *electro-optic* device, such as a *Pockels cell* is used to modulate a continuous light source, such as a CW laser or a xenon arc lamp. Alternatively, LEDs or laser diodes can be directly modulated.



In such a case, the excitation frequency is described by:

$$E(t) = E_0 [1 + M_E \sin \omega t]$$

$E(t)$  and  $E_0$  are the intensities at time  $t$  and  $0$ ,  $M_E$  is the modulation factor which is related to the ratio of the AC and DC parts of the signal and  $\omega$  is the angular modulation frequency.

$$\omega = 2\pi f \text{ where } f \text{ is the linear modulation frequency}$$

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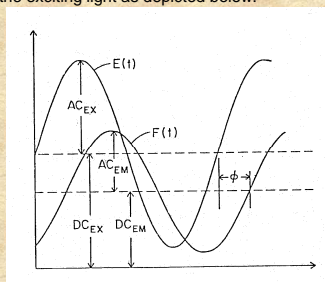
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Due to the persistence of the excited state, fluorophores subjected to such an excitation will give rise to a modulated emission which is shifted in phase relative to the exciting light as depicted below.



This sketch illustrates the phase delay ( $\phi$ ) between the excitation,  $E(t)$ , and the emission,  $F(t)$ . Also shown are the AC and DC levels associated with the excitation and emission waveforms.

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One can demonstrate that:

$$F(t) = F_0 [1 + M_F \sin(\omega t + \phi)]$$

This relationship signifies that measurement of the phase delay,  $\phi$ , forms the basis of one measurement of the lifetime,  $\tau$ . In particular one can demonstrate that:

$$\tan \phi = \omega \tau$$

The *modulations* of the excitation ( $M_E$ ) and the emission ( $M_F$ ) are given by:

$$M_E = \left( \frac{AC}{DC} \right)_E \quad \text{and} \quad M_F = \left( \frac{AC}{DC} \right)_F$$

The *relative modulation*,  $M$ , of the emission is then:

$$M = \frac{(AC/DC)_F}{(AC/DC)_E}$$

$\tau$  can also be determined from  $M$  according to the relation:  $M = \frac{1}{\sqrt{1 + (\omega \tau)^2}}$

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Using the *phase shift* and *relative modulation* one can thus determine a *phase lifetime* ( $\tau_P$ ) and a *modulation lifetime* ( $\tau_M$ ).

If the fluorescence decay is a single exponential, then  $\tau_P$  and  $\tau_M$  will be equal at all modulation frequencies.

If, however, the fluorescence decay is multiexponential then  $\tau_P < \tau_M$  and, moreover, the values of both  $\tau_P$  and  $\tau_M$  will depend upon the modulation frequency, i.e.,

$$\tau_P(\omega_1) < \tau_P(\omega_2) \quad \text{if} \quad \omega_1 > \omega_2$$

To get a feeling for typical phase and modulation data, consider the following data set.

Frequency (MHz)	$\tau_P$ (ns)	$\tau_M$ (ns)
5	6.76	10.24
10	6.02	9.70
30	3.17	6.87
70	1.93	4.27

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These differences between  $\tau_P$  and  $\tau_M$  and their frequency dependence form the basis of the methods used to analyze for lifetime heterogeneity, i.e., the component lifetimes and amplitudes.

In the case just shown, the actual system being measured was a mixture of two fluorophores with lifetimes of 12.08 ns and 1.38 ns, with relative contributions to the total intensity of 53% and 47% respectively.

Here must be careful to distinguish the term *fractional contribution to the total intensity* (usually designated as  $f$ ) from  $\alpha$ , the pre-exponential term referred to earlier. The relation between these two terms is given by:

$$f_i = \frac{\alpha_i \tau_i}{\sum_j \alpha_j \tau_j}$$

where  $j$  represents the sum of all components. In the case just given then, the ratio of the pre-exponential factors corresponding to the 12.08 ns and 1.38 ns components is approximately 1/3. In other words, there are three times as many molecules in solution with the 1.38 ns lifetime as there are molecules with the 12.08 ns lifetime.

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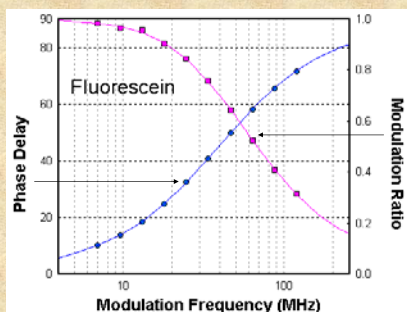
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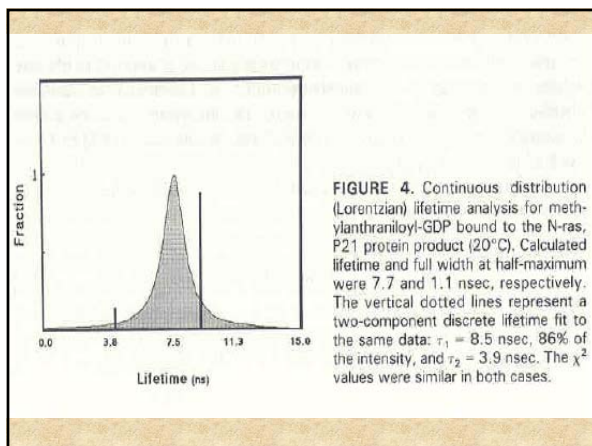
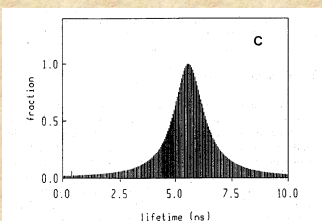
Multifrequency phase and modulation data are usually presented as shown below:



The plot shows the frequency response curve (phase and modulation) of Fluorescein in phosphate buffer pH 7.4 acquired on an ISS Chronos using a 470 nm LED. The emission was collected through a 530 high pass filter. The data is best fitted by a single exponential decay time of 4 ns.

In addition to decay analysis using discrete exponential decay models, one may also choose to fit the data to *distribution* models. In this case, it is assumed that the excited state decay characteristics of the emitting species actually results in a large number of lifetime components. Shown below is a typical lifetime distribution plot for the case of single tryptophan containing protein – human serum albumin.

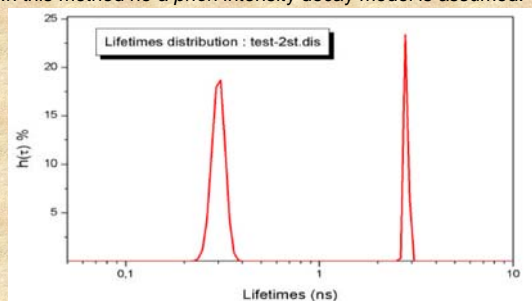
The distribution shown here is Lorentzian but depending on the system different types of distributions, e.g., Gaussian or asymmetric distributions, may be utilized. This approach to lifetime analysis is described in: Alcalá, J. R., E. Gratton and F. G. Prendergast. Fluorescence lifetime distributions in proteins. *Biophys. J.* 51, 597-604 (1987).



**FIGURE 4.** Continuous distribution (Lorentzian) lifetime analysis for methylantraniloyl-GDP bound to the N-ras, P21 protein product (20°C). Calculated lifetime and full width at half-maximum were 7.7 and 1.1 nsec, respectively. The vertical dotted lines represent a two-component discrete lifetime fit to the same data:  $\tau_1 = 8.5$  nsec, 86% of the intensity, and  $\tau_2 = 3.9$  nsec. The  $\chi^2$  values were similar in both cases.



Another popular lifetime analysis method – based on Information Theory - is the *Maximum Entropy Method* (MEM). In this method no *a priori* intensity decay model is assumed.



Jean-Claude Brochon  
Maximum entropy method of data analysis in time-resolved spectroscopy.  
Methods Enzymol. 1994;240:262-311.

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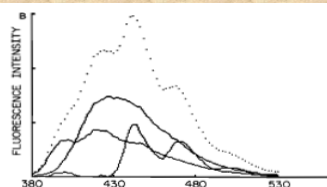
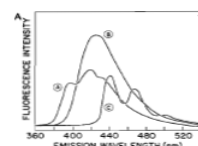
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Wavelength dependent lifetime data can be used to resolve individual spectra in a mixture

Gratton, E. and Jameson, D.M. (1985)  
*Anal. Chem.* 57:1694-1697. New  
Approach to Phase and Modulation  
Resolved Spectra.

Mixture: Lifetimes were 10.8ns, 4.3ns and 0.9ns

Spectra of individual components



**Figure 3.** (A) Intensity spectra of individual components, POPOP (A), DENS (B), and perylene (C) in ethanol (not degassed). (B) Intensity (dotted line) and phase resolved spectra (solid lines) for the ternary mixture in A.

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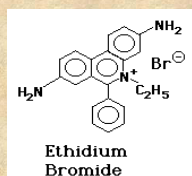
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### Global Analysis

In Global Analysis one can link decay parameters across many data sets which often allows for a more robust analysis

**Example of the application of Global Analysis**

Binding of Ethidium-Bromide to Transfer RNA



Yeast phenylalanine tRNA




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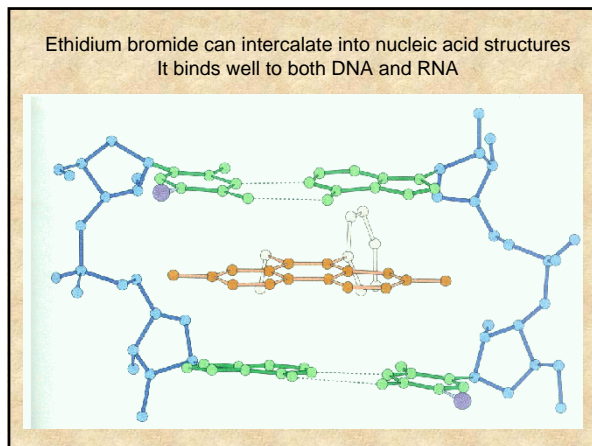
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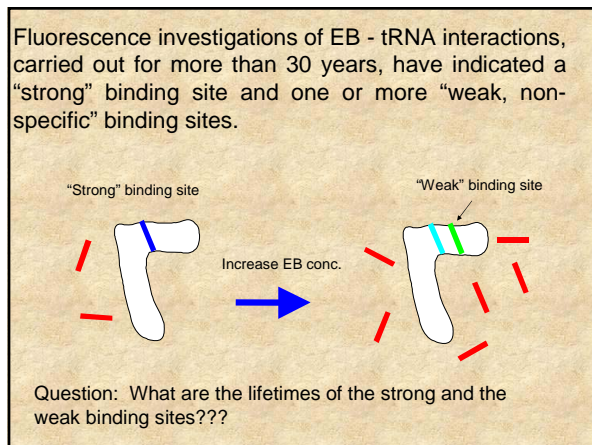
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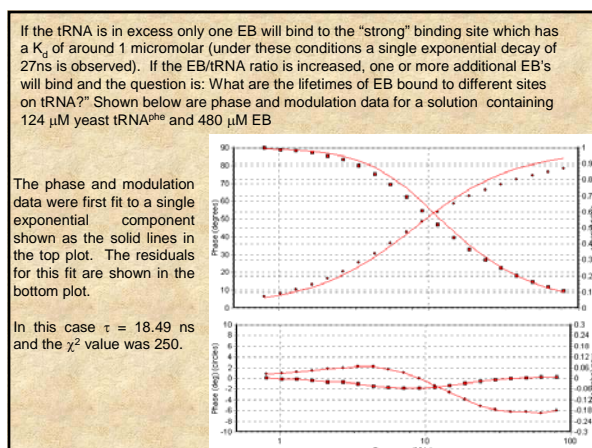
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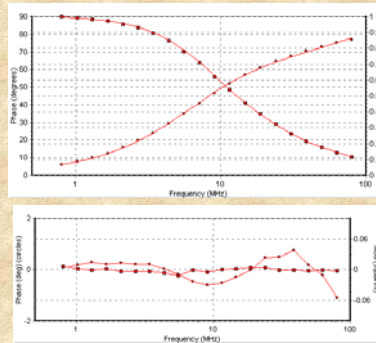
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The data were then fit to a 2-component model shown here. In this case the two lifetime components were 22.71 ns with a fractional intensity of 0.911 and 3.99 ns with a fractional intensity of 0.089.

The  $\chi^2$  for this fit was 3.06 (note the change in scale for the residual plot compared to the first case shown).




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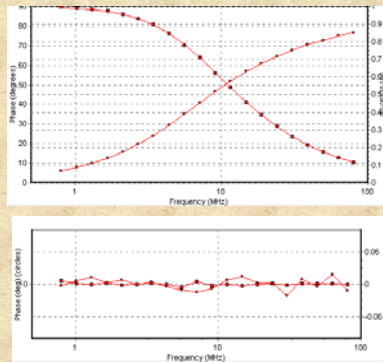
A 3-component model improves the fit still more. In this case

$$\tau_1 = 24.25 \text{ ns}, f_1 = 0.83$$

$$\tau_2 = 8.79 \text{ ns}, f_2 = 0.14$$

$$\tau_3 = 2.09 \text{ ns}, f_3 = 0.03$$

$$\chi^2 = 0.39.$$




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Adding a fourth component – with all parameters free to vary - does not lead to a significant improvement in the  $\chi^2$ . In this case one finds 4 components of 24.80 ns (0.776), 12.13ns (0.163), 4.17 ns (0.53) and 0.88 ns (0.008).

But we are not using all of our information! We can actually fix some of the components in this case. We know that **free EB** has a lifetime of **1.84 ns** and we also know that the lifetime of **EB bound to the “strong” tRNA binding site** is **27 ns**. So we can fix these in the analysis. The results are four lifetime components of 27 ns (0.612), 18.33 ns (0.311), 5.85 ns (0.061) and 1.84 ns (0.016). The  $\chi^2$  improves to 0.16.

We can then go one step better and carry out “**Global Analysis**”. In Global Analysis, multiple data sets are analyzed simultaneously and different parameters (such as lifetimes) can be “linked” across the data sets. The important concept in this particular experiment is that the lifetimes of the components stay the same and only their fractional contributions change as more ethidium bromide binds.

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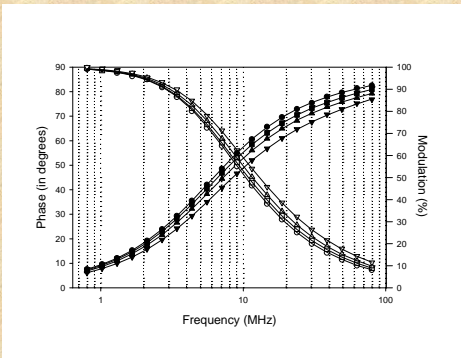
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In this system, 8 data sets, with increasing EB/tRNA ratios, were analyzed. Some of the data are shown below for EB/tRNA ratios of 0.27 (circles), 1.34 (squares), 2.41 (triangles) and 4.05 (inverted triangles).



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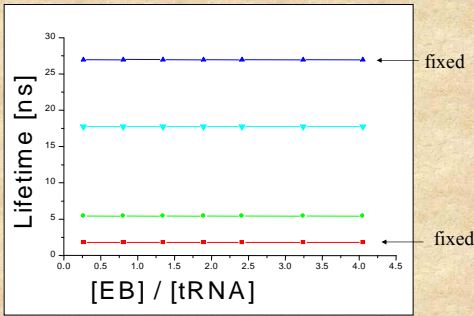
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Global Analysis on seven data sets fit best to the 4 component model with two fixed components of 27ns and 1.84ns and two other components of 17.7ns and 5.4ns.



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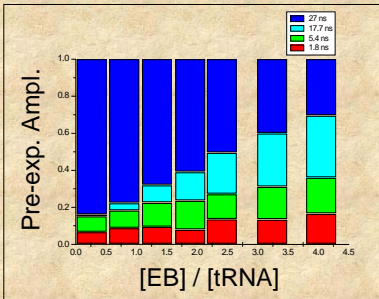
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As shown in the plot below, as the EB/tRNA ratio increases the fractional contribution of the 27ns component decreases while the fractional contributions of the 17.7ns and 5.4ns components increase.



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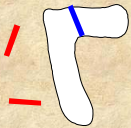
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### The Model

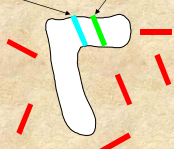
"Strong" binding site  
Lifetime ~ 27ns



→ Increase EB conc. →

Lifetime decrease  
To 17.7ns

"Weak" binding site  
Lifetime ~5.4ns



Question:  
Is the drop in the lifetime of the "strong" binding site due to a change in tRNA conformation or energy transfer???

Answer: ???

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Later in this workshop you'll learn about Fluorescence Lifetime Imaging or FLIM

In FLIM, lifetime data are obtained through a microscope - lifetime data is acquired at each pixel in the image

Lifetime data is more robust than intensity since it does not depend on how many fluorescent molecules are present

**A: Single photon FLIM**

	GFP intensity	GFP lifetime
GFP-Cdk42 control		
GFP-Cdk42/ WT PAK-myc-Cy3 +EGF		
GFP-N17Cdk42/ WT PAK-myc-Cy3 +EGF		

**B: Multi photon FLIM**

	Multi-photon GFP intensity	GFP lifetime
GFP-Cdk42 control		
GFP-Cdk42/ WT PAK-myc-Cy3 +EGF		
GFP-N17Cdk42/ WT PAK-myc-Cy3 +EGF		

B. D. Venetta, Rev. Sci. Instrum., 1959, 30, 450-457.

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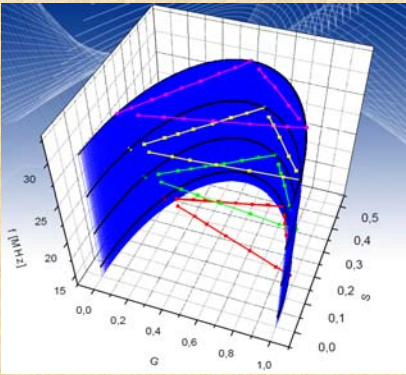
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### Introduction to Phasors



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Urbana 2012

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Registered from The Journal of Physical Chemistry, 1981, 85, 498.  
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## Resolution of the Fluorescence Lifetimes in a Heterogeneous System by Phase and Modulation Measurements

Gregorio Weber  
Department of Biochemistry, School of Chemical Sciences, University of Illinois, Urbana, Illinois 61801 (Received August 12, 1980)

A closed-form procedure is described for the determination of the decay constants and the relative contributing intensities of the  $N$  independent components of a heterogeneous fluorescence emission employing measurements of the phase shift and relative modulation of the total fluorescence at  $N$  appropriate harmonic excitation frequencies. At each frequency the phase and modulation measurements yield the real part of the Fourier transform of the fluorescence impulse response,  $G$ , and its imaginary part,  $S$ . It is shown that the moments of a distribution of the lifetimes are linear combinations of the  $G$ s (zero and even moments) or the  $S$ s (odd moments), and the rule for the construction of the coefficients of  $G$  and  $S$  in these linear combinations is derived. The classical deconvolution method is used to obtain the lifetimes and fractional contributions of the components from the moments. For binary and ternary mixtures the numerical complications required are trivial. In the present state of the art, the lifetimes of the components of a binary mixture should be derivable with a loss in precision somewhat smaller than 1 order of magnitude with respect to the overall measured lifetimes.

$$G_c = M_c \cos \phi_c = [(1 + (\omega_c \tau_c^2)^2)^{-1/2} + (\omega_c \tau_c^2)^2]^{1/2} \quad (9)$$

$$S_c = M_c \sin \phi_c = G_c \omega_c \tau_c^2 \quad (10)$$

$$G(\omega) = \int_0^\infty f(t) \cos \omega t \, dt$$

$$S(\omega) = - \int_0^\infty f(t) \sin \omega t \, dt \quad (5)$$

APPLIED SPECTROSCOPY REVIEWS, VOL. 15-106 (1984)

## The Measurement and Analysis of Heterogeneous Emissions by Multifrequency Phase and Modulation Fluorometry

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### APPENDIX 3. PHASE AND MODULATION LIFTING RELATIONS

We have asserted that a heterogeneous emitting population, in the absence of excited state reactions, will demonstrate a phase lifetime which is always less than the modulation lifetime. The algebraic demonstration of this fact is somewhat cumbersome (11, 49). We present here a brief and more intuitive demonstration of the phenomenon.

One may make a single geometrical representation of the phase delay and relative modulation as shown in Fig. 11. Here we depict a vector of length  $M$  making an angle  $\phi$  with the  $x$ -axis where  $\phi$  represents the phase delay and  $M$  the relative modulation. Since for a single exponential decay we have the relation  $M = \cos \phi$ , the endpoint of the vector is constrained to lie on the circle of radius  $1/2$  with a center at  $(1/2, 0)$ . The intercept of the extension of this vector with the line through  $x = 1$  equals of course  $\tau = \omega^{-1}$ . This circle is identical for single exponential systems irrespective of the lifetime or modulation frequency. We note that the  $G$  and  $S$  intercepts of the vector correspond to our previously defined  $G$  and  $S$  functions (since  $G = M \cos \phi$  and  $S = M \sin \phi$ ).

Figure 12 represents the case of two exponential decays with phase delays  $\phi_1$  and  $\phi_2$  and relative modulations of  $M_1$  and  $M_2$ , respectively. These decays contribute to the total observed lifetime decay with fractional weights of  $f_1$  and  $f_2$ , respectively. The total fluorescence observed is represented by the vector sum,  $M$ , of the two components and gives an observed phase delay of  $\phi$ . Again we see that the intercept of the extension of the  $M$  vector with the  $x = 1$  line corresponds to  $\tau^{-1}$  (since  $\tau = \omega^{-1}$ ). The value of  $\omega^2 \tau^2$ , however, corresponds to the line segment  $AB$ . This observation follows from the fact that the triangle  $AOB$ ,

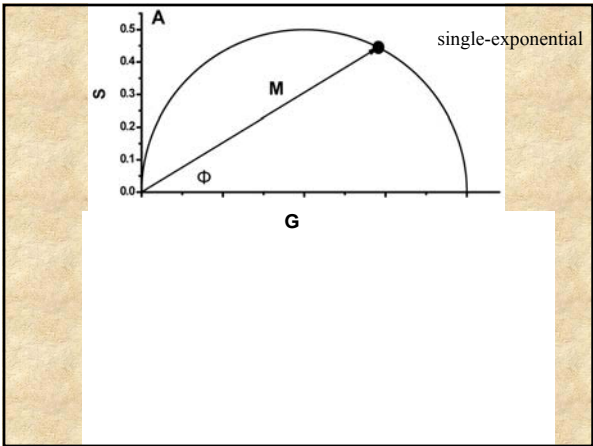
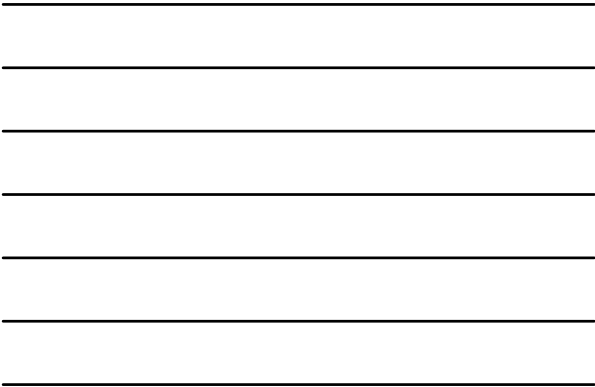
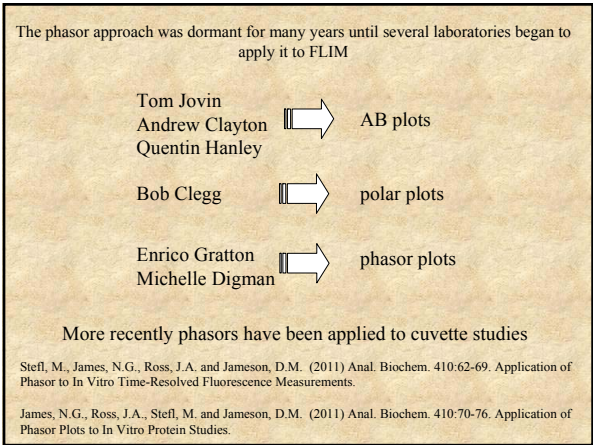
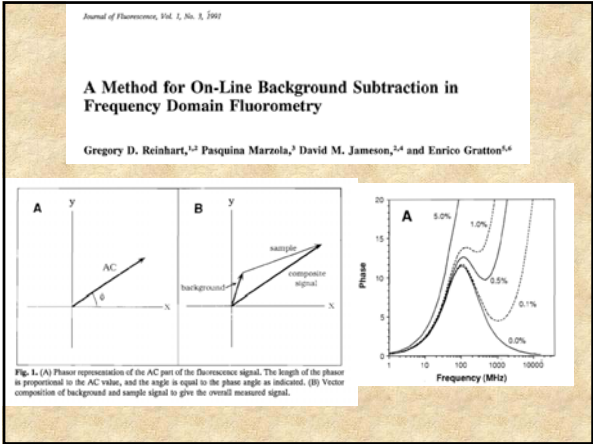
FIG. 11. Geometrical representation of phase delay ( $\phi$ ) and modulation ratio ( $M$ ) for a single exponential decay.

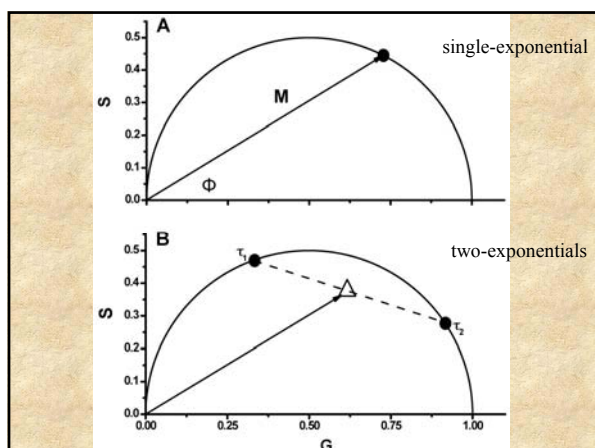
98 JAMESON, GRATTON, AND HALL

FIG. 12. Geometrical representation of phase delay ( $\phi$ ) and modulation ratio ( $M$ ) for a double exponential decay.

The mathematical equivalence of impulse and harmonic response data does not, however, mean that practical differences between the methods do not exist. One important difference which is seldom mentioned concerns the fact that the impulse response technique does not permit the analysis of photons originating over the complete time domain. The time domain data are in essence truncated and obliged to be between somewhat arbitrary limits in addition to falling into time bins of finite width. In the harmonic response, regardless of the modulating frequency, the time information content of the emitted photons is not sacrificed. Moreover, in the single photon counting pulse technique one is permitted to observe at best only one photon per exciting pulse (to avoid statistical pileup effects this limit is, in fact, reduced) whereas in the harmonic response approach all the photons contribute to the measured signal. This latter fact confers a sensitivity to the harmonic method which compares favorably with that of single photon counting, contrary to popular belief.








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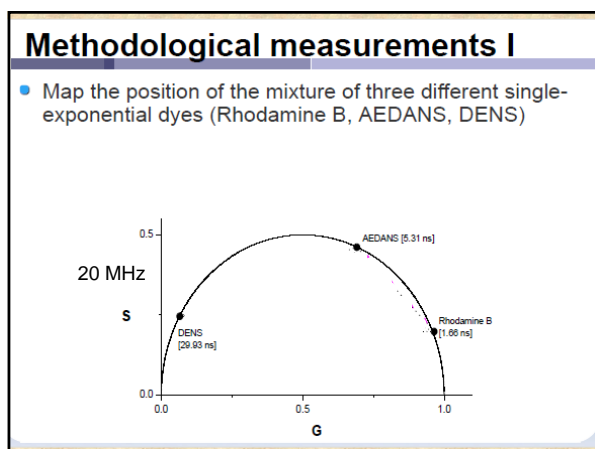
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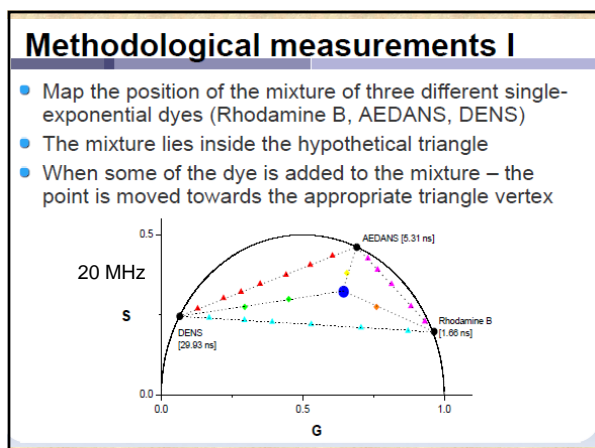
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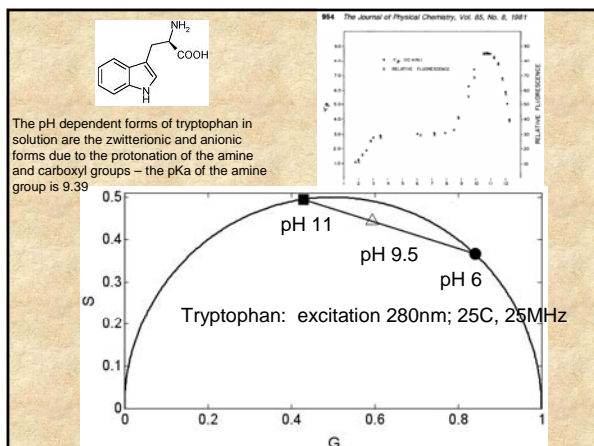
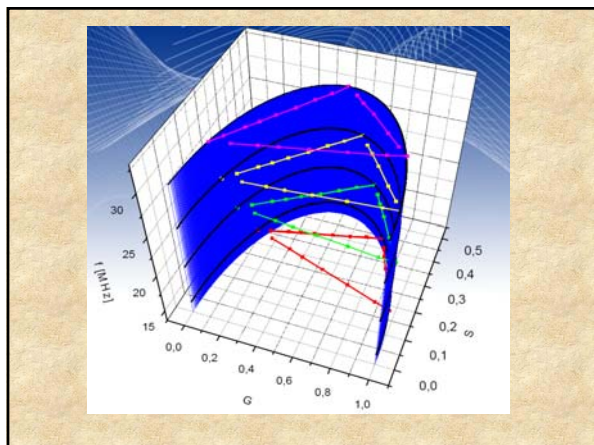
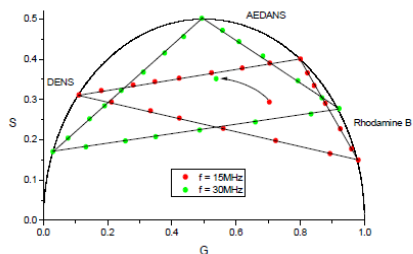
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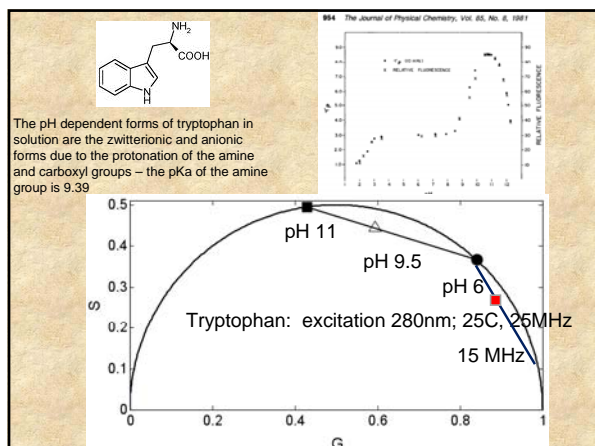
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## Methodological measurements II

- When analyzed for different frequencies – position of the hypothetical triangle in universal circle is anticlockwise shifted when modulation frequency is increased
- Middle points copy the curvature of the universal circle





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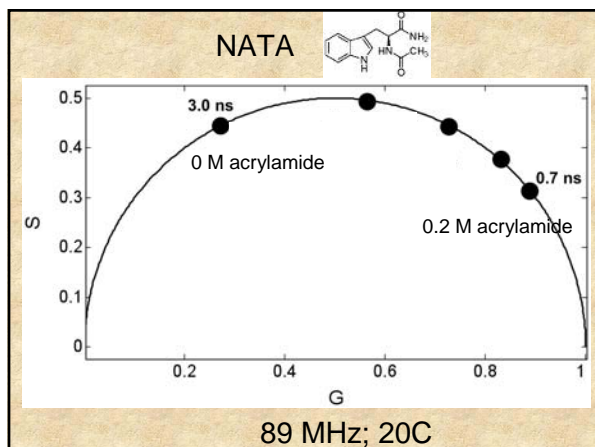
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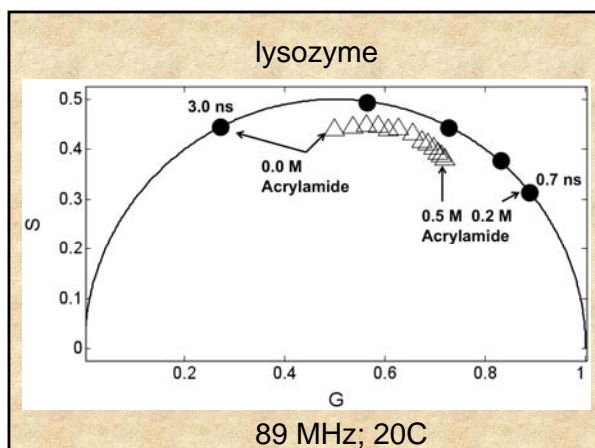
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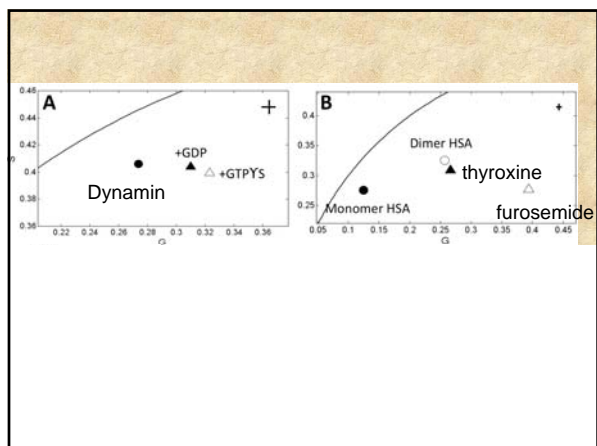
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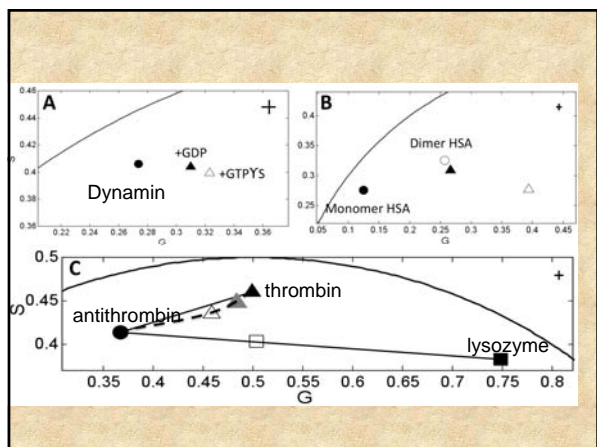
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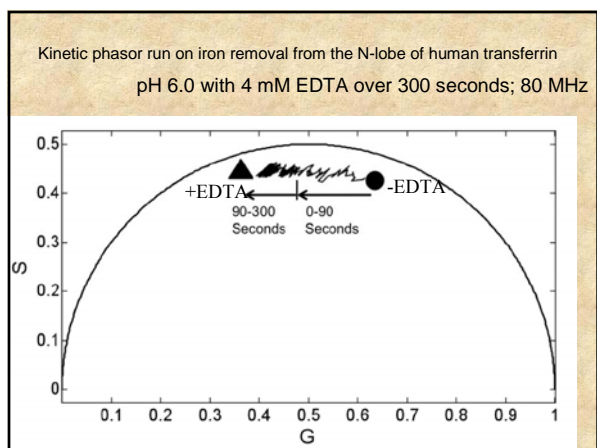
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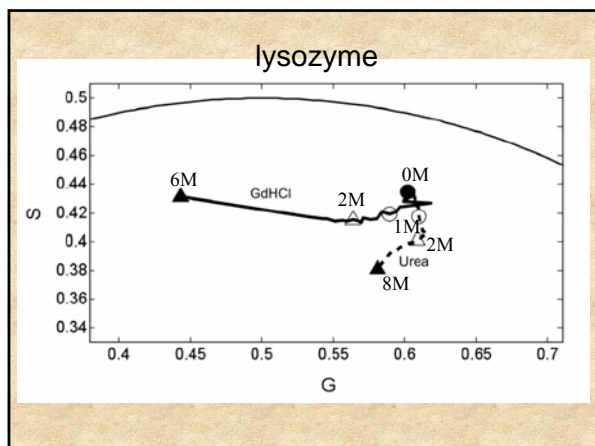
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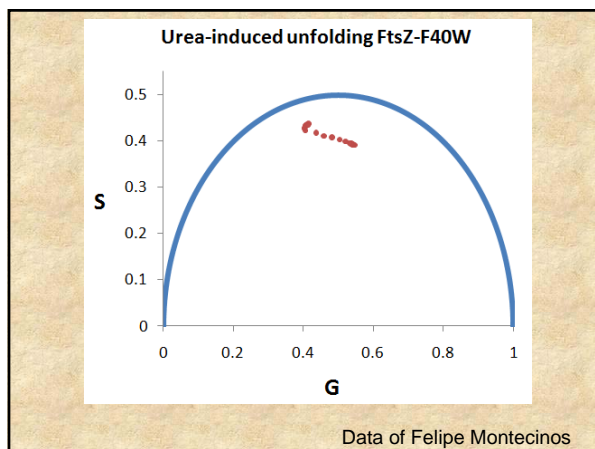
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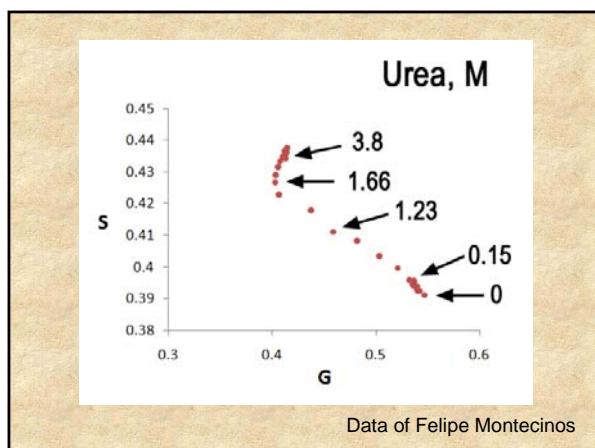
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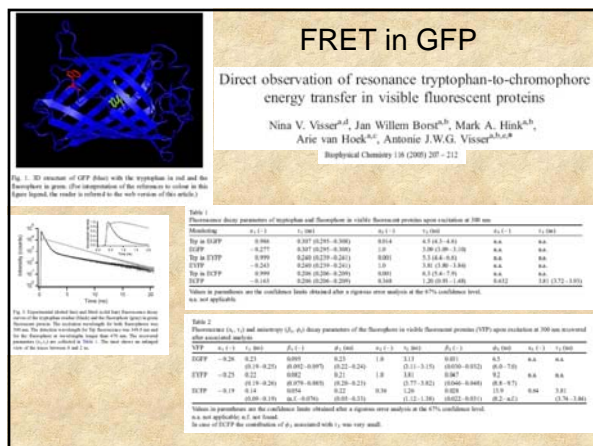
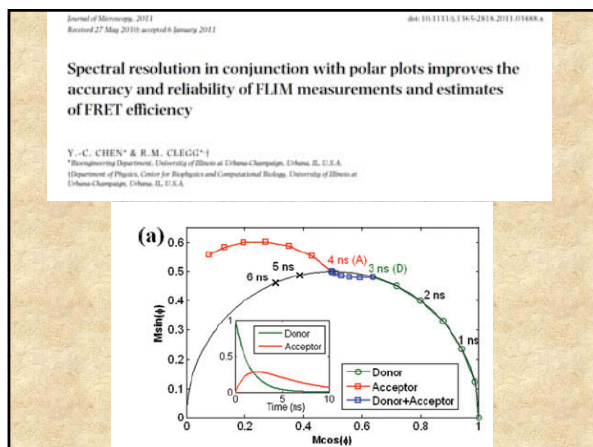
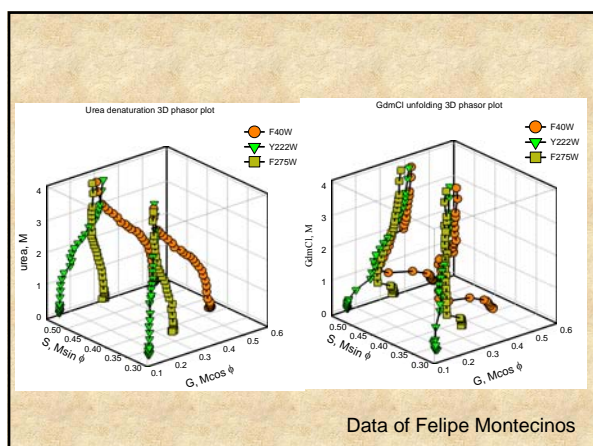
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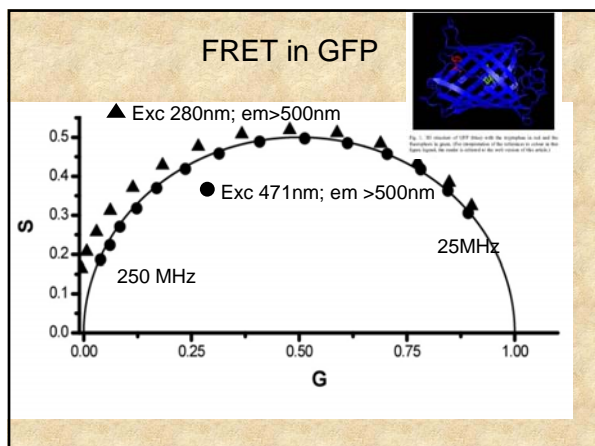
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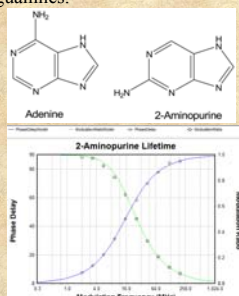
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G-Quadruplex DNA are four stranded structures containing two or more stacked square-planar G-tetrads, composed of four Hoogsteen hydrogen bonded guanines.



2-Aminopurine free in aqueous solution gives a single exponential lifetime of ~10 ns.

G-Quadruplex structure and stability illuminated by phasor plots  
Robert Buscaglia, David M. Jameson and Jonathan B. Chaires  
*Nucleic Acid Research* In Press

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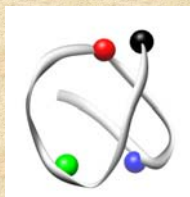
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2-aminopurine Bases in Human Telomere Quadruplex Structures  
– Unique Environments for Each Position

1AP - 5'-XGGGTTAGGGTTAGGGTTAGGG  
7AP - 5'-AGGGTTXGGGTTAGGGTTAGGG  
13AP - 5'-AGGGTTAGGGTTXGGGTTAGGG  
19AP - 5'-AGGGTTAGGGTTAGGGTTXGGG




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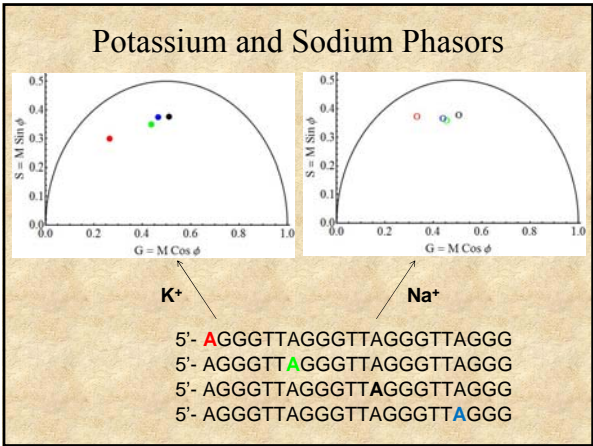
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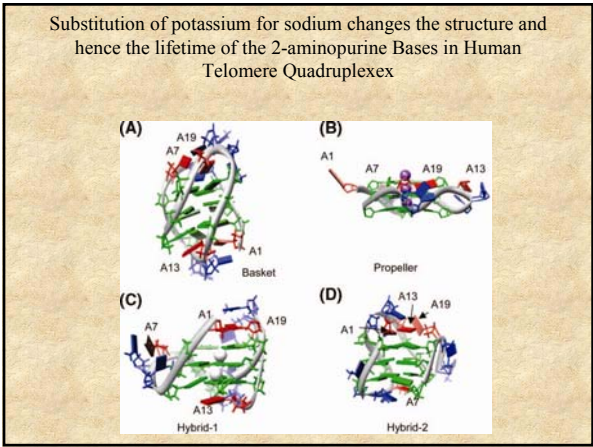
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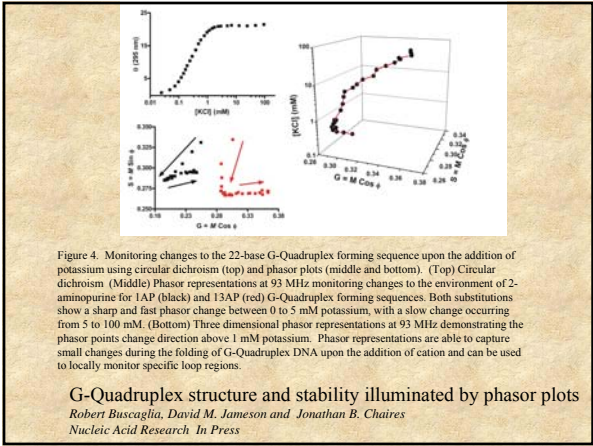
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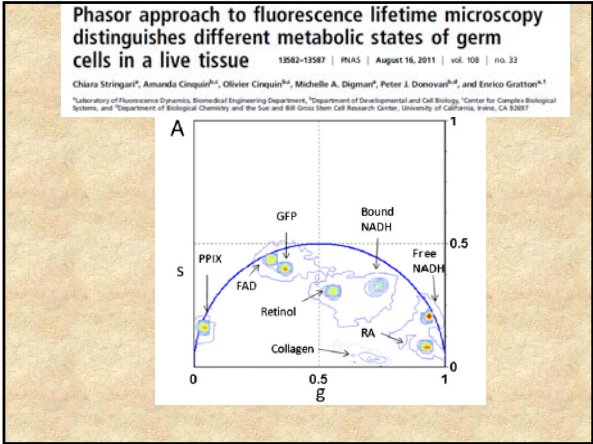
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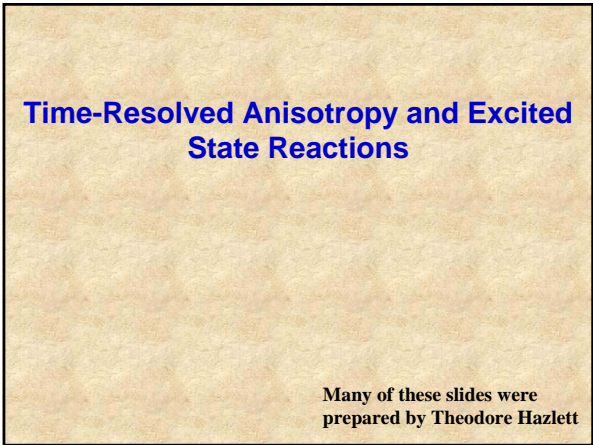
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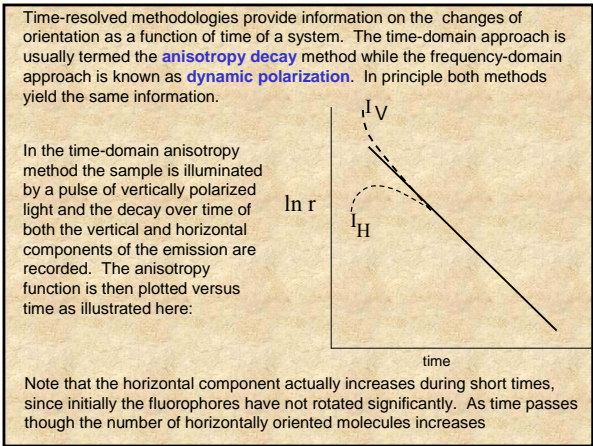
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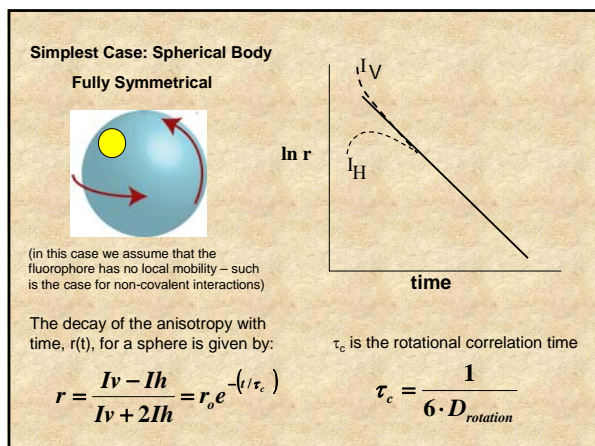
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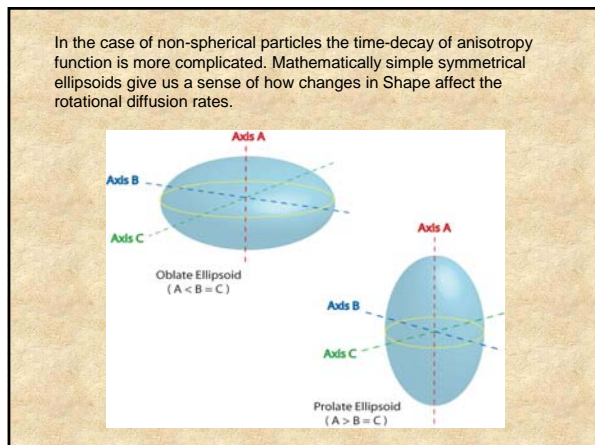
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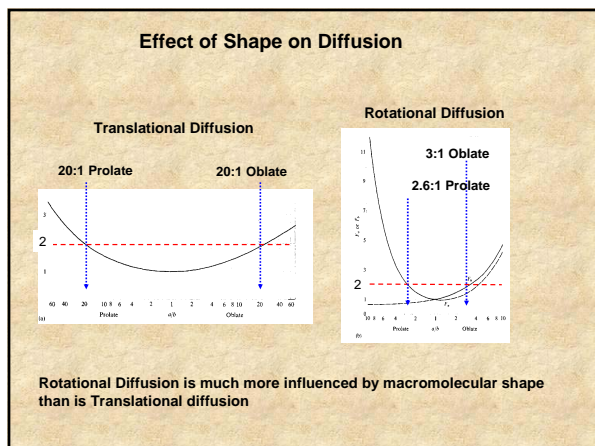
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### How are these Shapes Modeled?

In the case of symmetrical ellipsoids of revolution the relevant expression is:

$$\mathbf{r}(t) = r_1 e^{\left(\frac{-t}{\tau_{c1}}\right)} + r_2 e^{\left(\frac{-t}{\tau_{c2}}\right)} + r_3 e^{\left(\frac{-t}{\tau_{c3}}\right)}$$

where:  $\tau_{c1} = 1/6D_2$

$\tau_{c2} = 1/(5D_2 + D_1)$

$\tau_{c3} = 1/(2D_2 + 4D_1)$

$D_1$  and  $D_2$  are the rotational diffusion coefficients about the axes of symmetry and about either equatorial axis, respectively.

Resolution of the rotational rates is limited in practice to two rotational correlation times which differ by at least a factor of two.

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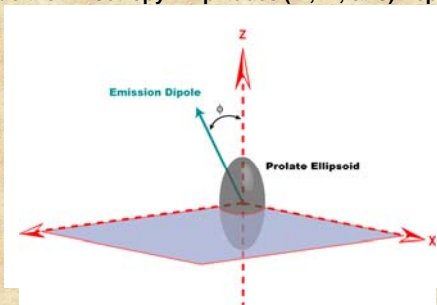
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### What do the Anisotropy Amplitudes ( $r_1$ , $r_2$ , & $r_3$ ) Represent?



The amplitudes relate to orientation of the probe with respect to the axis of symmetry for the ellipsoid (we are assuming colinear excitation and emission dipoles).

$$\begin{aligned} r_1 &= 0.1(3\cos^2\phi - 1)^2 \\ r_2 &= 0.3\sin^2(2\phi) \\ r_3 &= 0.3\sin^4(\phi) \end{aligned}$$

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### Multiple Rotating Species (mixtures)

$$\mathbf{r}(t) = r_1 e^{\left(\frac{-t}{\tau_{c1}}\right)} + r_2 e^{\left(\frac{-t}{\tau_{c2}}\right)}$$




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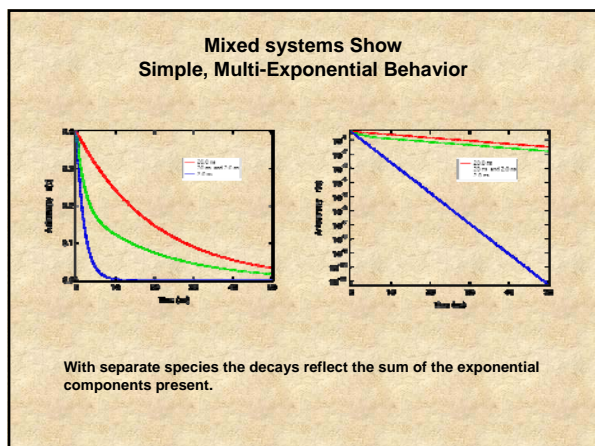
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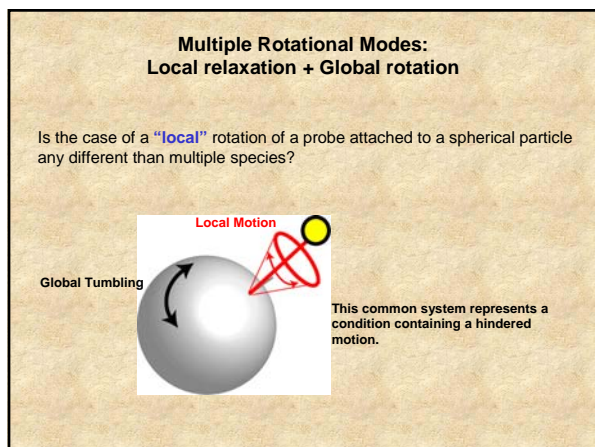
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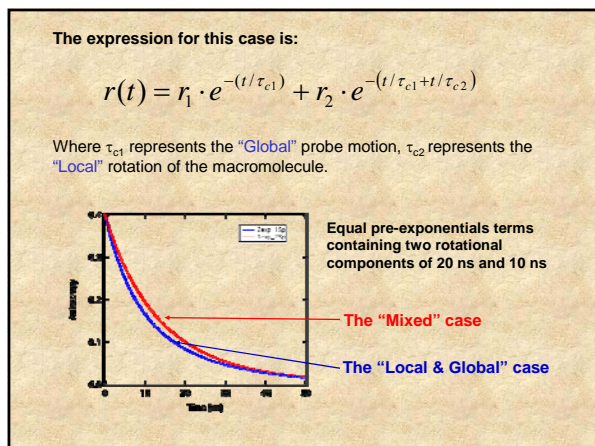
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### Hindered Rotational Systems Membrane Bilayers

Water molecules  
Fluorophore electric dipole

**Wobble-in-a-Cone Concept**

- 1) Freedom of motion
- 2) The rate of motion

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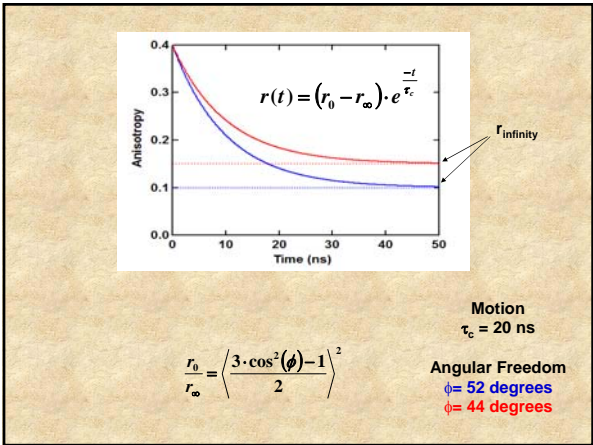
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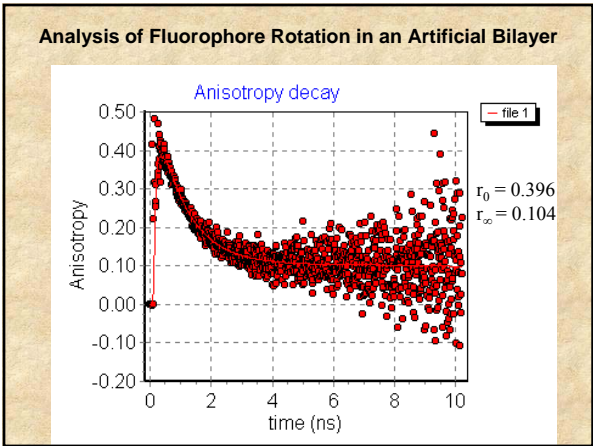
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### Phase & Modulation Measurements

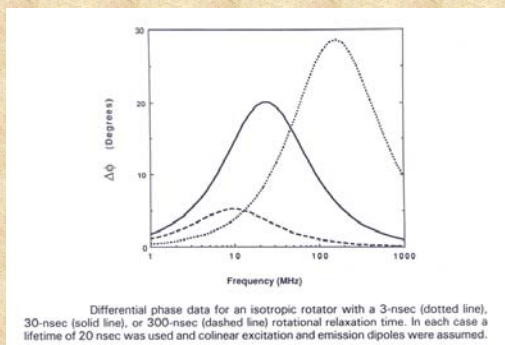
In **dynamic polarization measurements**, the sample is illuminated with vertically polarized, modulated light. The phase delay (dephasing) between the parallel and perpendicular components of the emission is measured as well as the modulation ratio of the AC contributions of these components. The expressions for a spherical particle are:

$$\Delta\phi = \tan^{-1} \left[ \frac{18\omega r_o R}{(k^2 + \omega^2)(1 + r_o - 2r_o^2) + 6R(6R + 2k + kr_o)} \right]$$

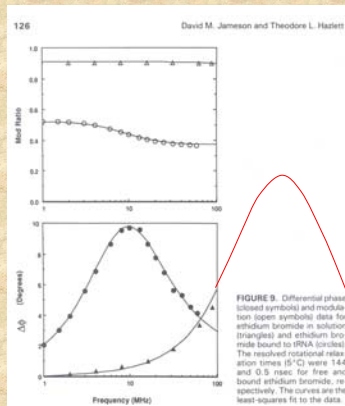
$$Y^2 = \frac{((1 - r_o)k + 6R)^2 + (1 - r_o)^2 \omega^2}{[(1 + 2r_o)k + 6R]^2 + (1 + 2r_o)^2 \omega^2}$$

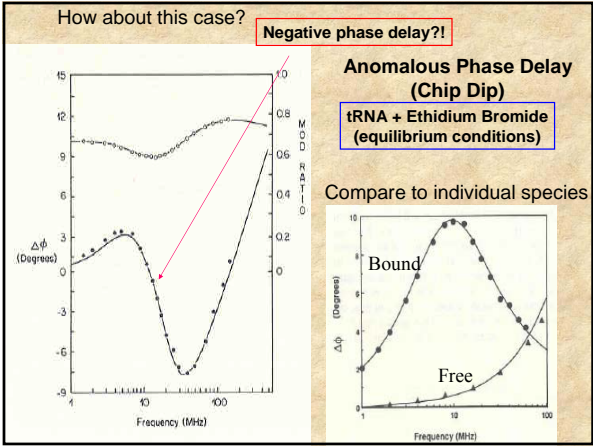
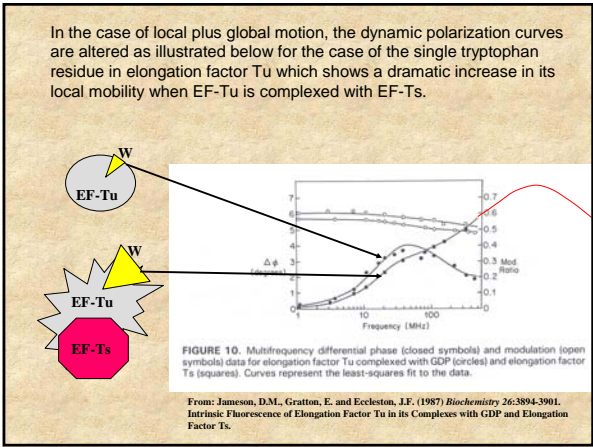
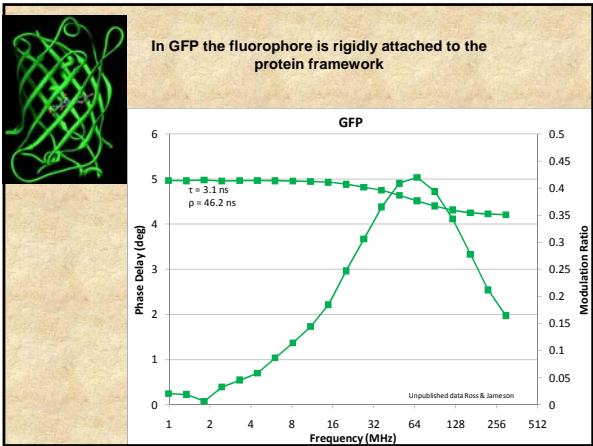
Where  $\Delta\phi$  is the phase difference, Y the modulation ratio of the AC components,  $\omega$  the angular modulation frequency,  $r_o$  the limiting anisotropy, k the radiative rate constant ( $1/\tau$ ) and R the rotational diffusion coefficient.

The illustration below depicts the  $\Delta\phi$  function for the cases of spherical particles with different rotational relaxation times.

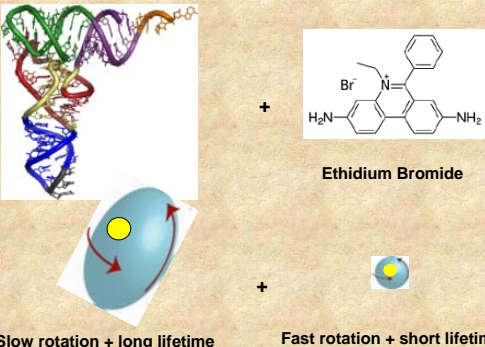


The figures here show actual results for the case of ethidium bromide free and bound to tRNA - one notes that the fast rotational motion of the free ethidium results in a shift of the "bell-shaped" curve to higher frequencies relative to the bound case. The lifetimes of free and bound ethidium bromide were approximately 1.8 ns and 27 ns respectively.





**What do We have in Our Mixture?**



**Ethidium Bromide**

Slow rotation + long lifetime      Fast rotation + short lifetime

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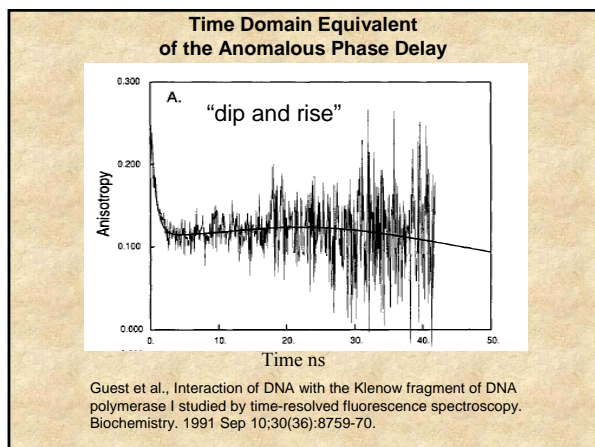
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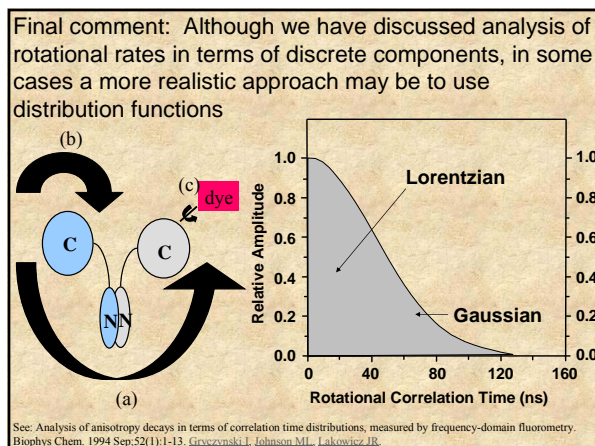
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## Quenching

A number of processes can lead to a reduction in fluorescence intensity, i.e., quenching

These processes can occur during the excited state lifetime – for example collisional quenching, energy transfer, charge transfer reactions or photochemistry – or they may occur due to formation of complexes in the ground state

We shall focus our attention on the two quenching processes usually encountered – namely collisional (dynamic) quenching and static (complex formation) quenching

### Collisional Quenching

Collisional quenching occurs when the excited fluorophore experiences contact with an atom or molecule that can facilitate non-radiative transitions to the ground state. Common quenchers include  $O_2$ ,  $I^-$ ,  $Cs^+$  and acrylamide.




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In the simplest case of collisional quenching, the following relation, called the **Stern-Volmer equation**, holds:

$$F_0/F = 1 + K_{SV}[Q]$$

where  $F_0$  and  $F$  are the fluorescence intensities observed in the absence and presence, respectively, of quencher,  $[Q]$  is the quencher concentration and  $K_{SV}$  is the **Stern-Volmer quenching constant**

In the simplest case, then, a plot of  $F_0/F$  versus  $[Q]$  should yield a straight line with a slope equal to  $K_{SV}$ .

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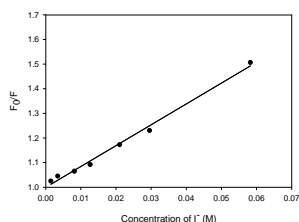
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Such a plot, known as a Stern-Volmer plot, is shown below for the case of fluorescein quenched by iodide ion ( $I^-$ ).



In this case,  $K_{SV} \sim 8 \text{ L}\cdot\text{mol}^{-1}$

$K_{SV} = k_q \tau_0$  where  $k_q$  is the **bimolecular quenching rate constant** (proportional to the sum of the diffusion coefficients for fluorophore and quencher) and  $\tau_0$  is the **excited state lifetime** in the absence of quencher.

In the case of purely collisional quenching, also known as **dynamic** quenching,:

$$F_0/F = \tau_0/\tau$$

Hence in this case:  $\tau_0/\tau = 1 + k_q \tau [Q]$

In the fluorescein/iodide system,  $\tau = 4 \text{ ns}$  and  $k_q \sim 2 \times 10^9 \text{ M}^{-1} \text{ sec}^{-1}$

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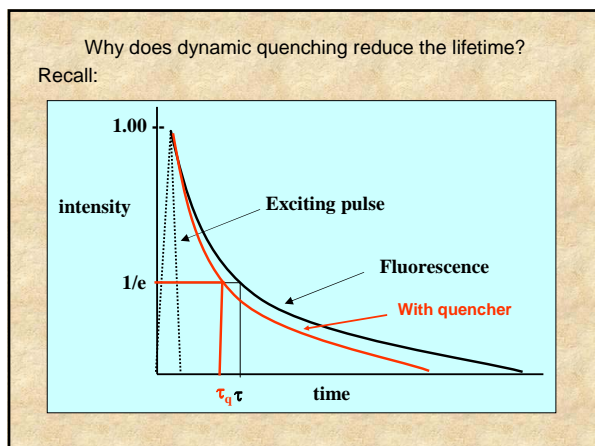
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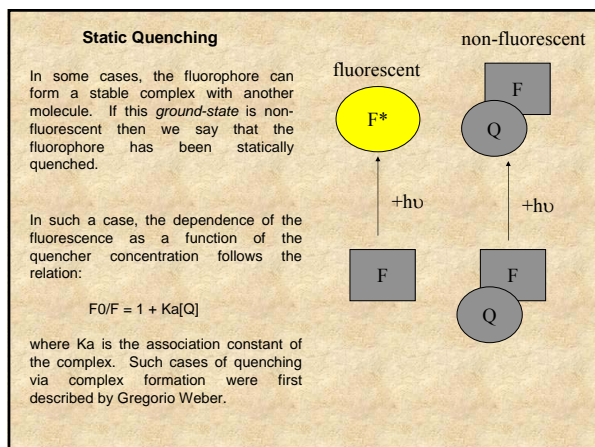
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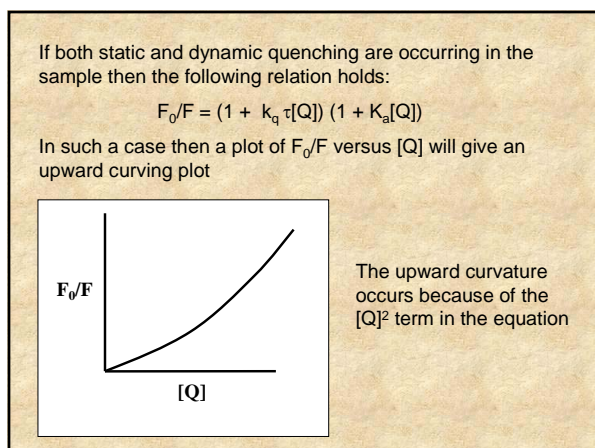
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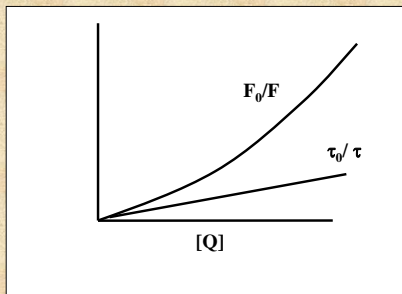
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However, since the lifetime is unaffected by the presence of quencher in cases of pure static quenching, a plot of  $\tau_0/\tau$  versus  $[Q]$  would give a straight line




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Sometimes you will see the equation for simultaneous static and dynamic quenching given as:

$$F_0/F = (1 + K_{SV}[Q])e^{V[Q]}$$

where the term  $e^{V[Q]}$  is used as a phenomenological descriptor of the quenching process. The term  $V$  in this equation represents an *active volume* element around the fluorophore such that any quencher within this volume at the time of fluorophore excitation is able to quench the excited fluorophore.

Non-linear Stern-Volmer plots can also occur in the case of purely collisional quenching if some of the fluorophores are less accessible than others. Consider the case of multiple tryptophan residues in a protein – one can easily imagine that some of these residues would be more accessible to quenchers in the solvent than other.

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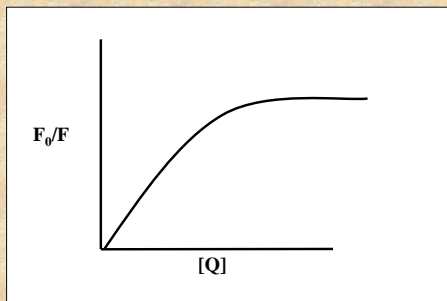
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In the extreme case, a Stern-Volmer plot for a system having accessible and inaccessible fluorophores could look like this:




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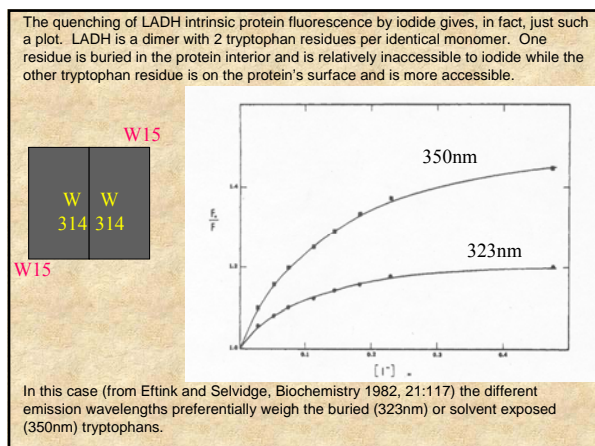
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